

# Learning Useful Representations of Recurrent Neural Network Weight Matrices

Vincent Herrmann, Francesco Faccio, Jürgen Schmidhuber

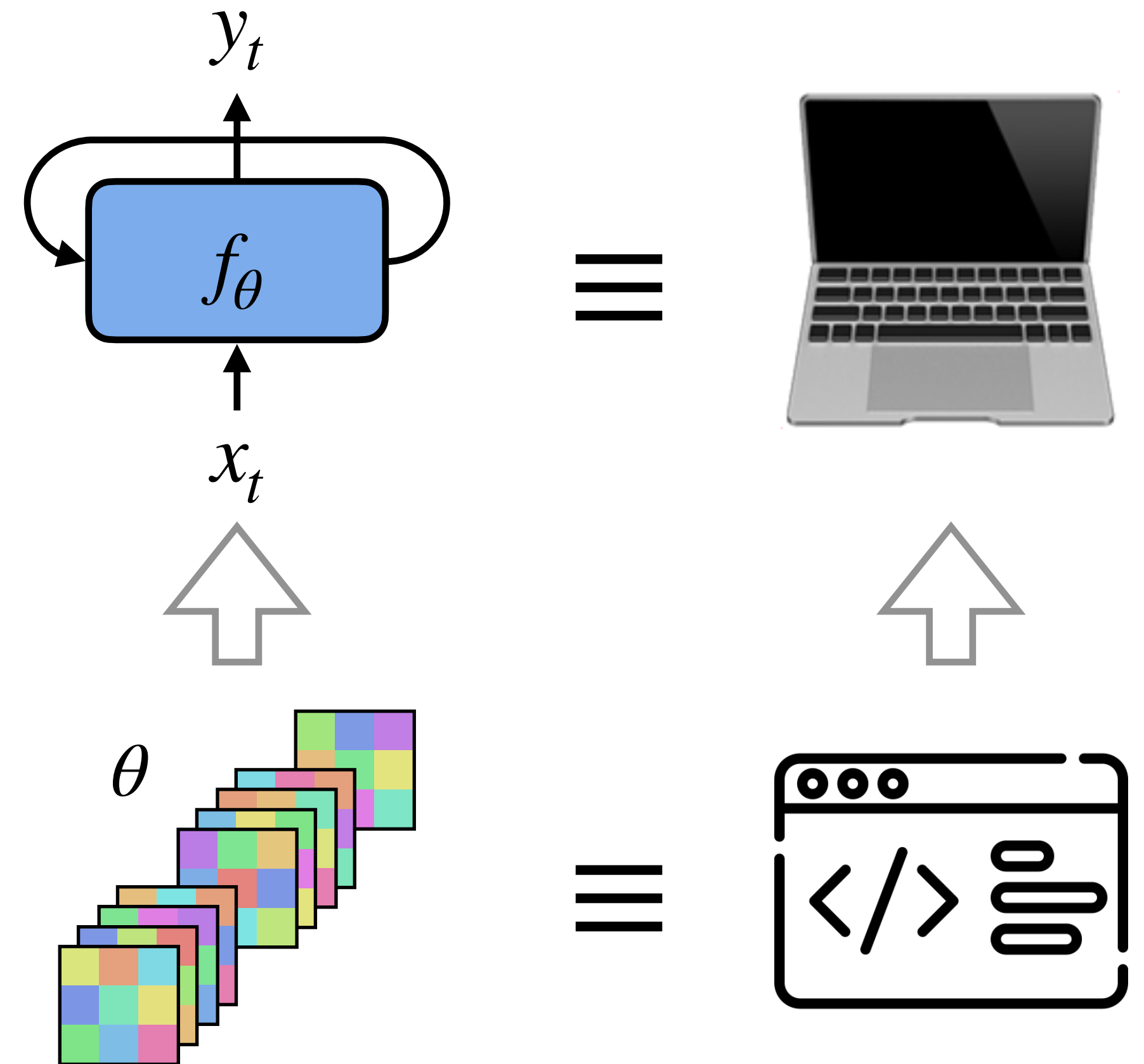


# Learning RNN Weight Representations

- RNNs are universal computers
- Weight matrices are their program

Applications:

- Reinforcement Learning
- Implicit Neural Representations
- Interpretability
- ...



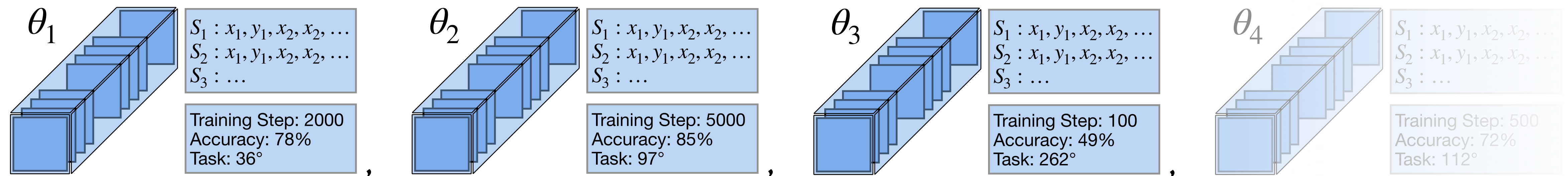
# Three Ingredients

- **Datasets**
- **Learning / Pre-Training method**
- **RNN Weight encoder architecture**

# Datasets

## Two RNN 'Model Zoos'

- 1000 LSTM training runs per dataset, 9 snapshots per run
- Each LSTM is trained on a different task from a task family
- Datapoint: LSTM weights  $\theta$ , 100 rollouts/trajectories, metadata



# Formal Languages Dataset

- Autoregressive models of 216 different formal languages
- Language family:  $L_{m_a, m_b, m_c, m_d} := \{a^{n+m_a}b^{n+m_b}c^{n+m_c}d^{n+m_d} \mid n \in \mathbb{N}\}$

- Examples

$L_{1,1,1,1} :$ 

a	b	c	d								
a	a	b	b	c	c	d	d				
a	a	a	b	b	b	c	c	c	d	d	d

  
 ...

$L_{1,2,1,2} :$ 

a	b	b	c	d	d								
a	a	b	b	b	c	c	d	d	d				
a	a	a	b	b	b	b	c	c	c	d	d	d	d

  
 ...

$L_{2,1,3,1} :$ 

a	a	b	c	c	c	d								
a	a	a	b	b	c	c	c	c	d	d				
a	a	a	a	b	b	b	c	c	c	c	c	d	d	d

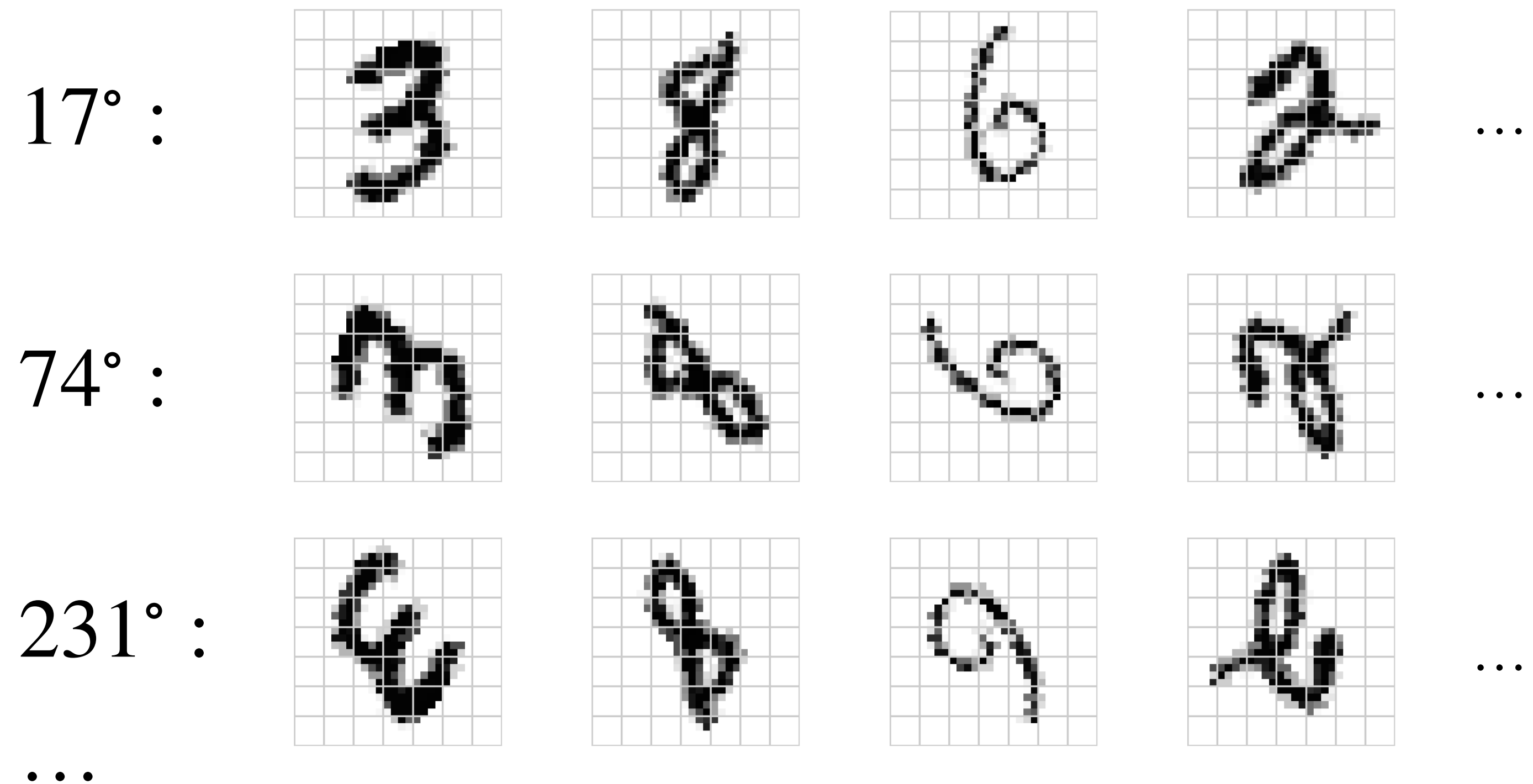
  
 ...

...

# Tiled Sequential MNIST Dataset

- Classifiers of tiled sequentialised MNIST digits (prediction at every step)
- Every model is trained on a different rotation angle of the digits

- Examples

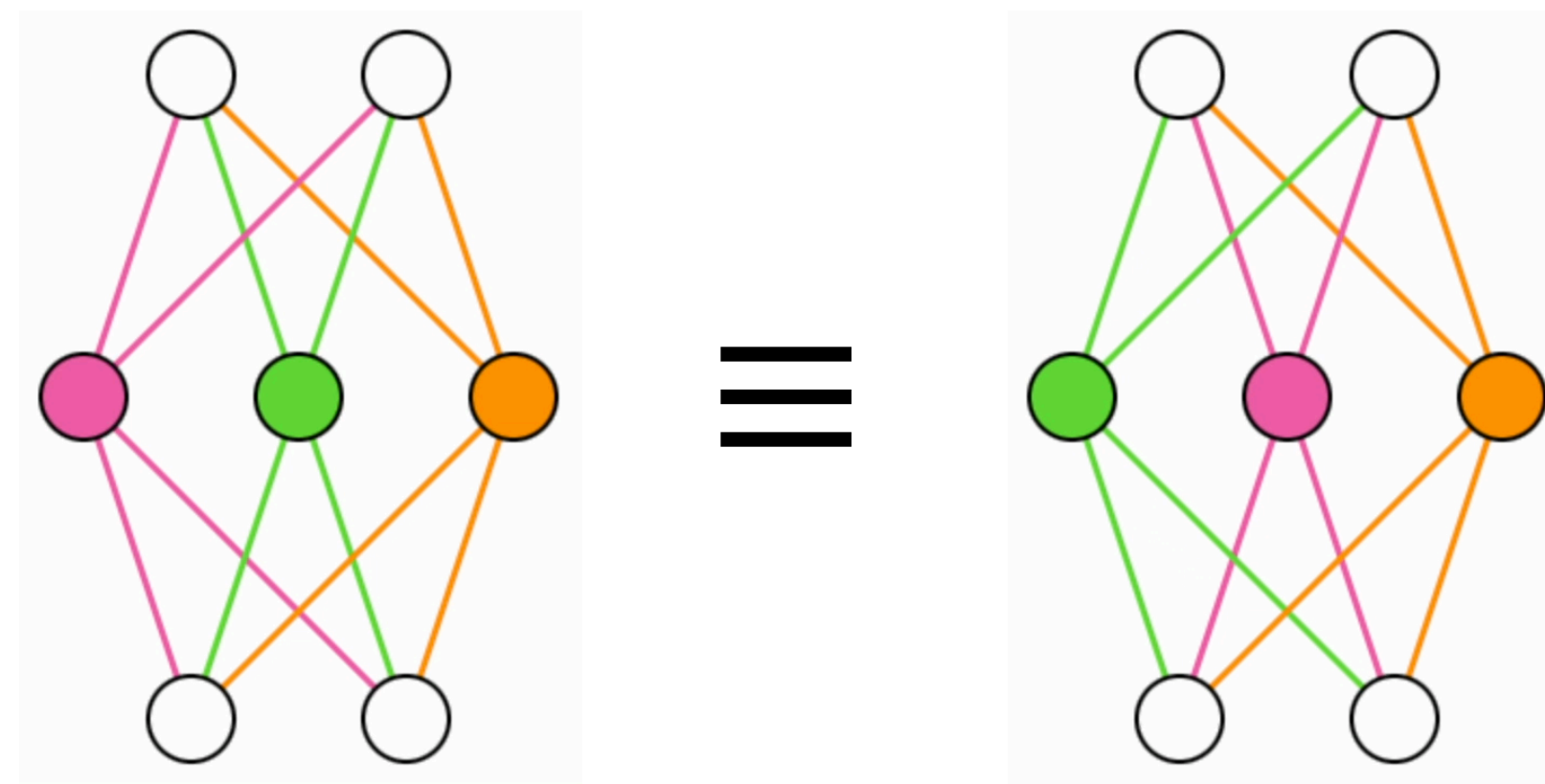


# Weight Space Symmetries

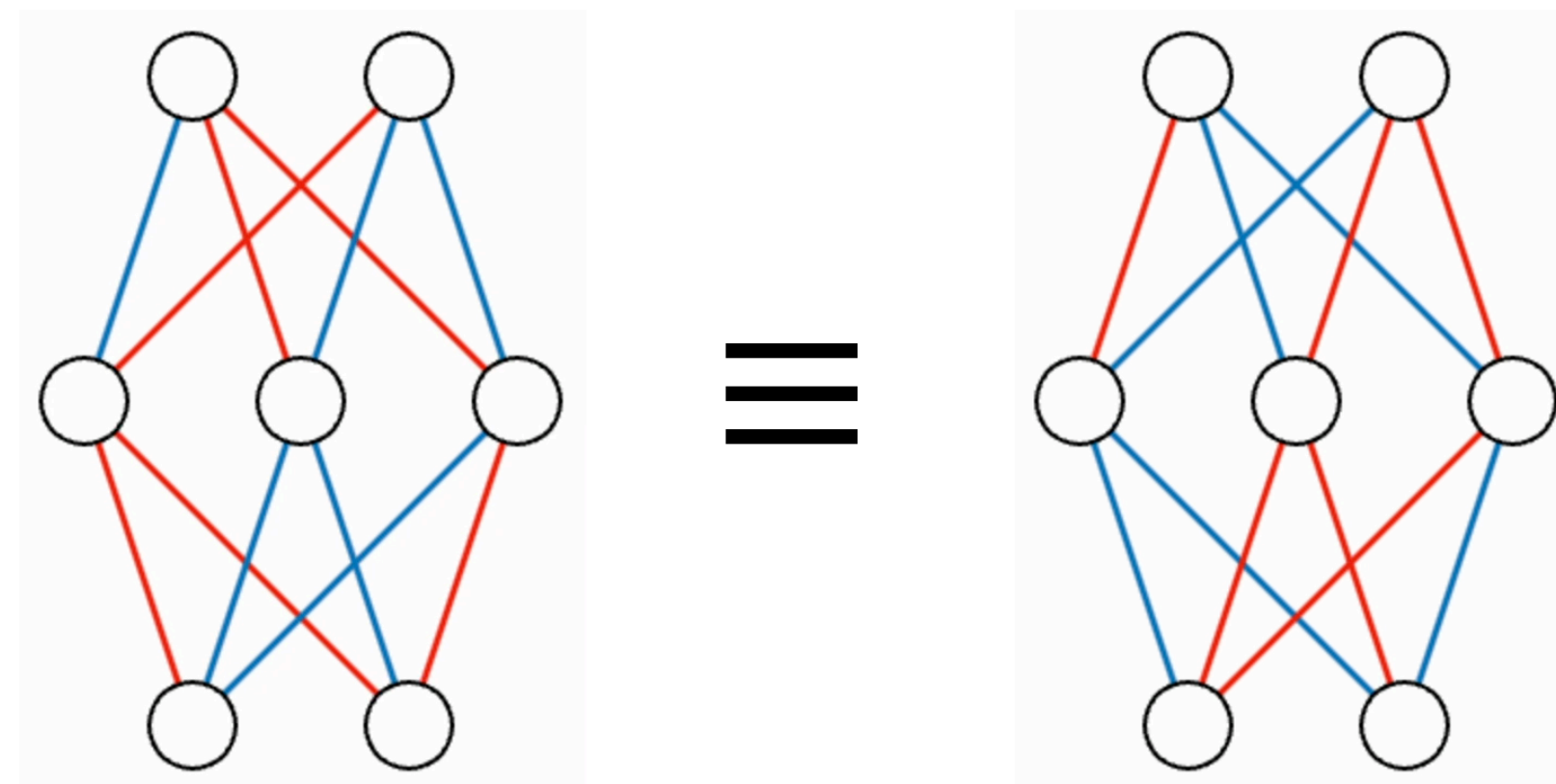
## Challenge and Opportunity

Huge number of equivalent networks

- Hidden Neuron Permutation

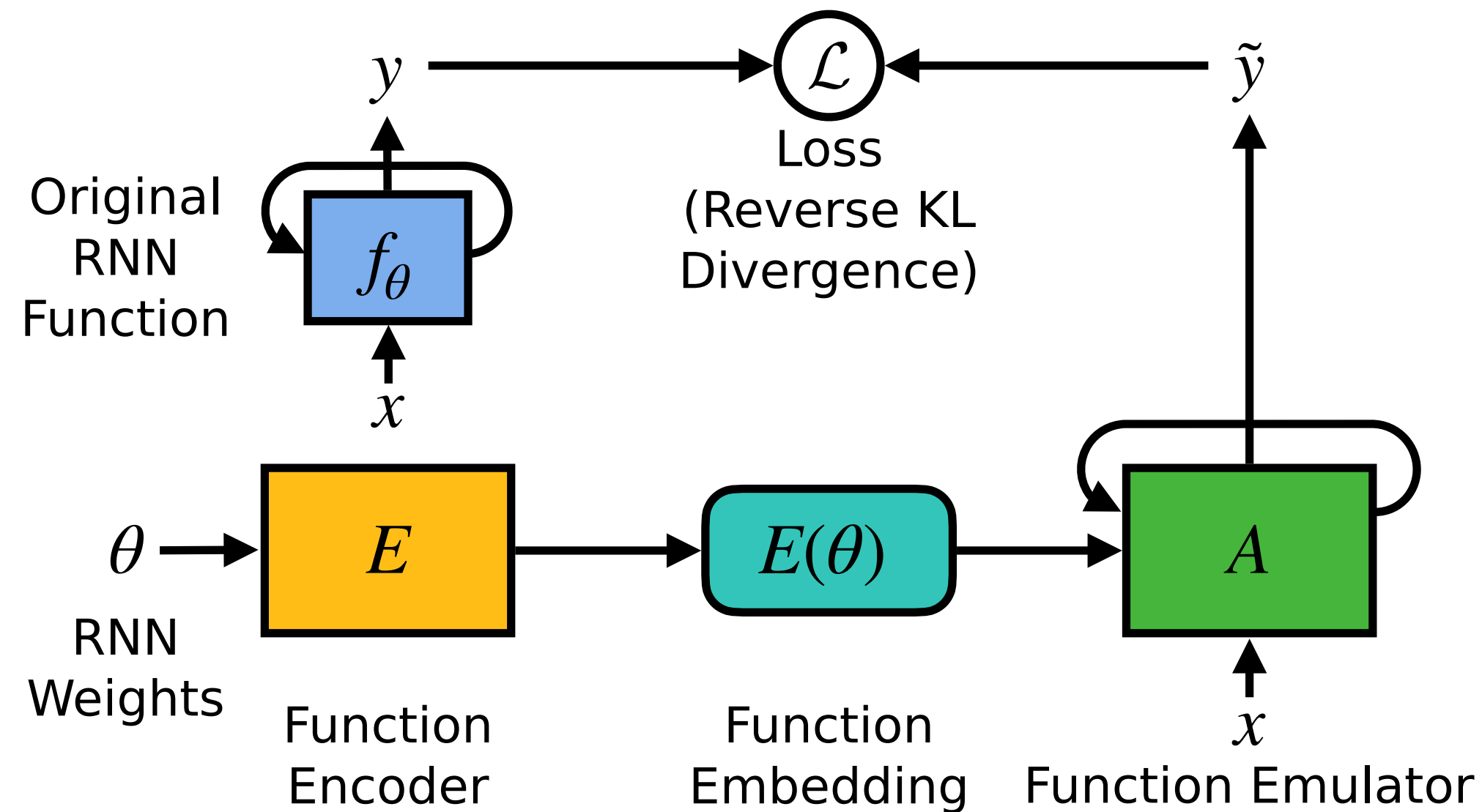


- Sign Flip / Scaling  
(depends on non-linearity)



# Self-Supervised Training Method

- Reconstruction or naive contrastive methods? 🚫 *Weight Space Symmetries* 🚫
- *Instead*: Emulate the functionality of the network!



- Encoder  $E$  generates representation  $E(\theta)$  of RNN  $f_\theta$
- Emulator  $A$  is conditioned on  $E(\theta)$  and imitates  $f_\theta$

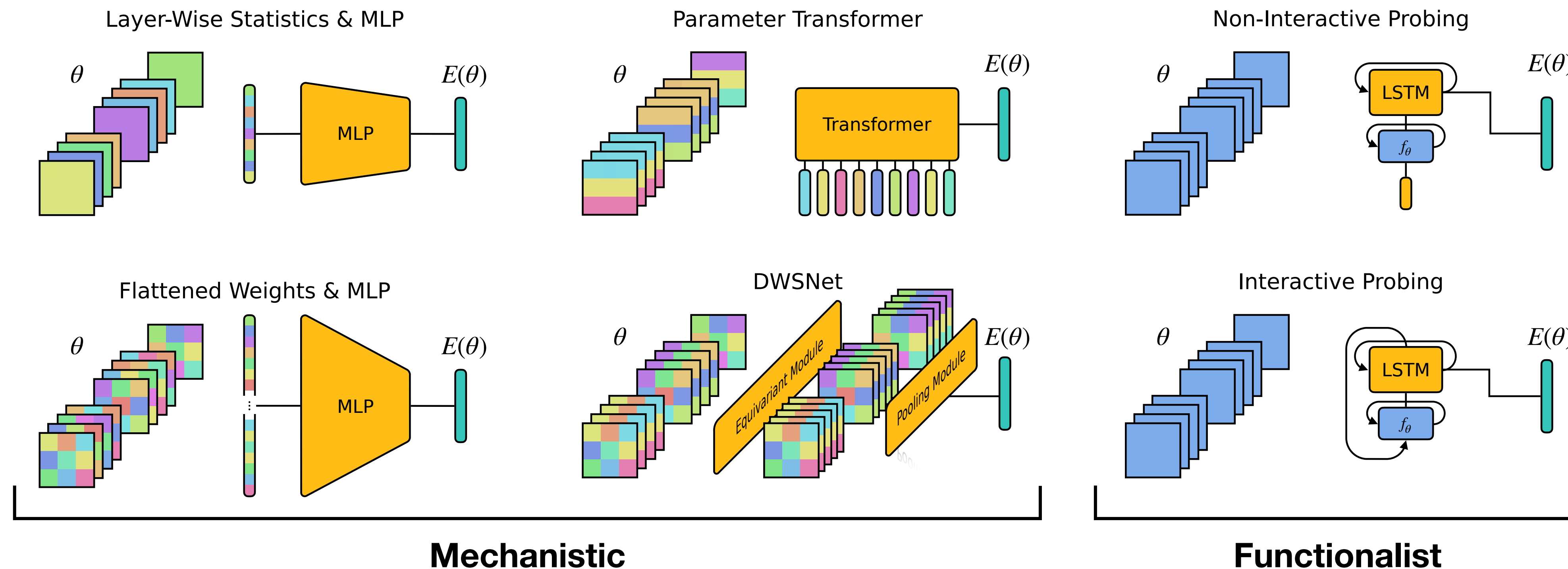
compare:

Raileanu et al., 2020, "Fast Adaptation via Policy-Dynamics Value Functions"



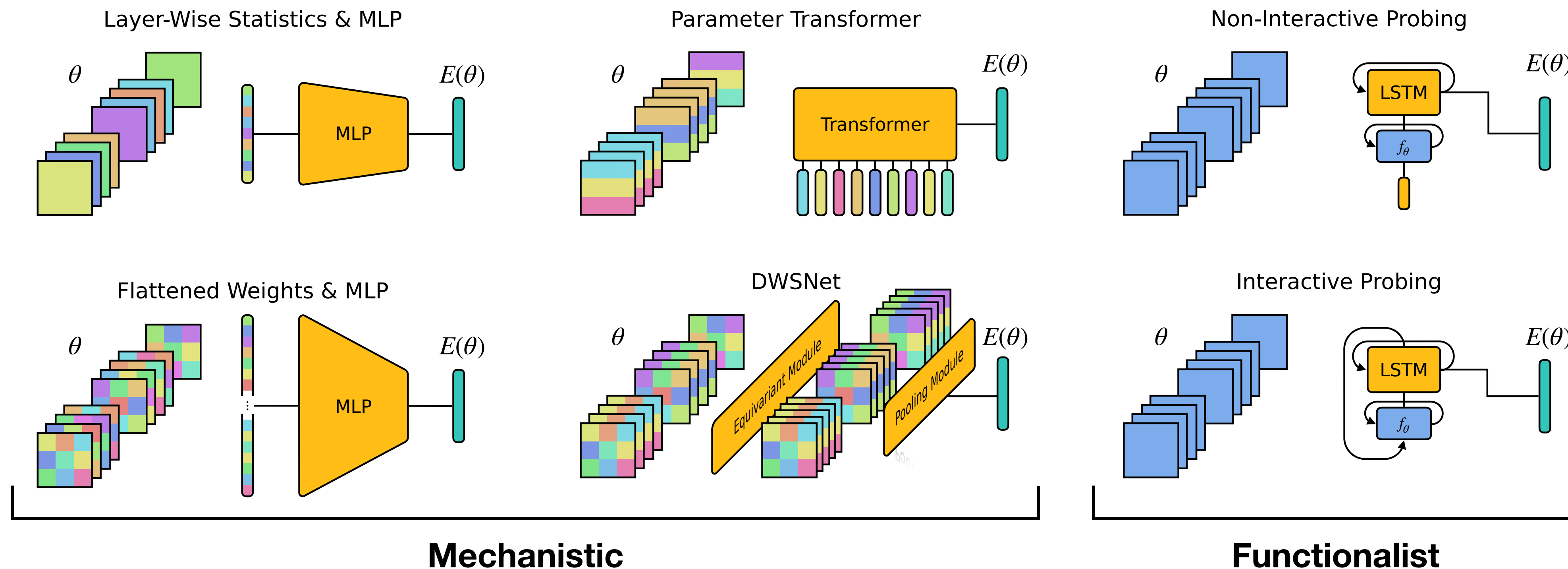
# RNN Weight Encoder Architectures

- Distinction: Mechanistic Encoders & Functionalist Encoders
- **Mechanistic** encoders look at the weights  $\theta$  directly
- **Functionalist** encoders look at the input-output mapping of  $f_\theta$



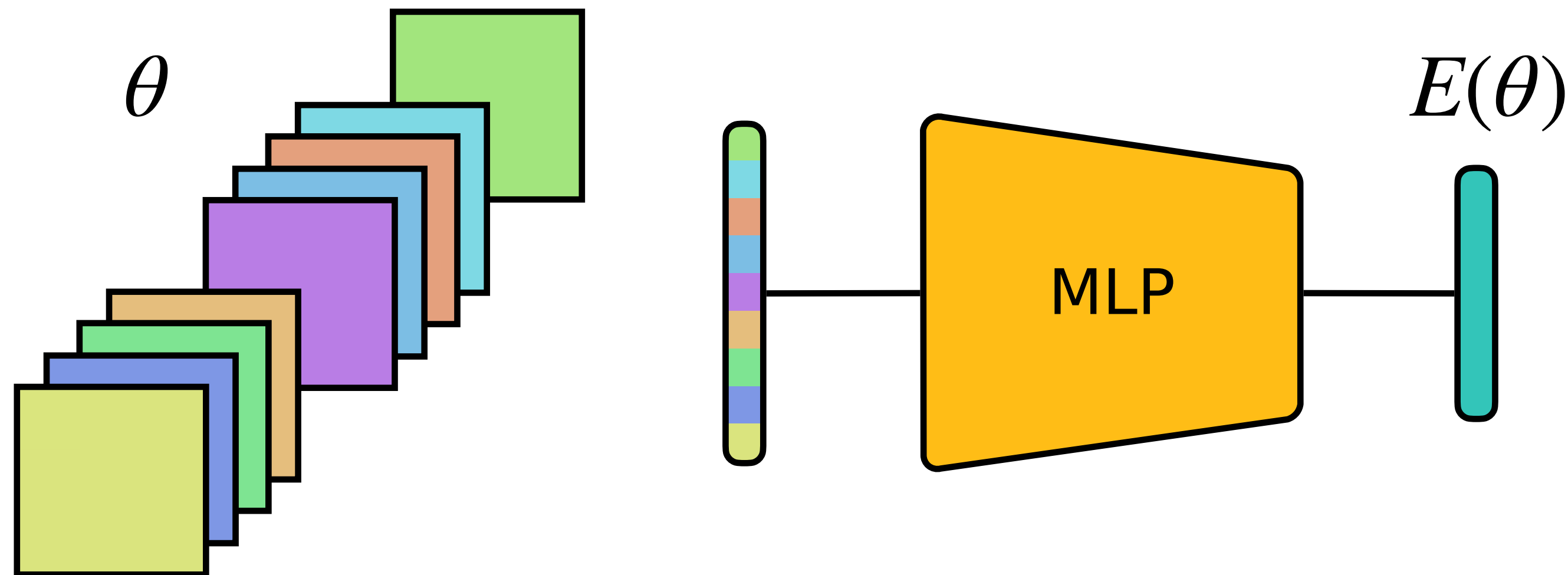
# Layer-Wise Statistics & MLP (Mechanistic)

- Concatenate various global statistics of each weight matrix and feed into MLP
- Invariant to hidden neuron permutation 👍
- Not universal 👎



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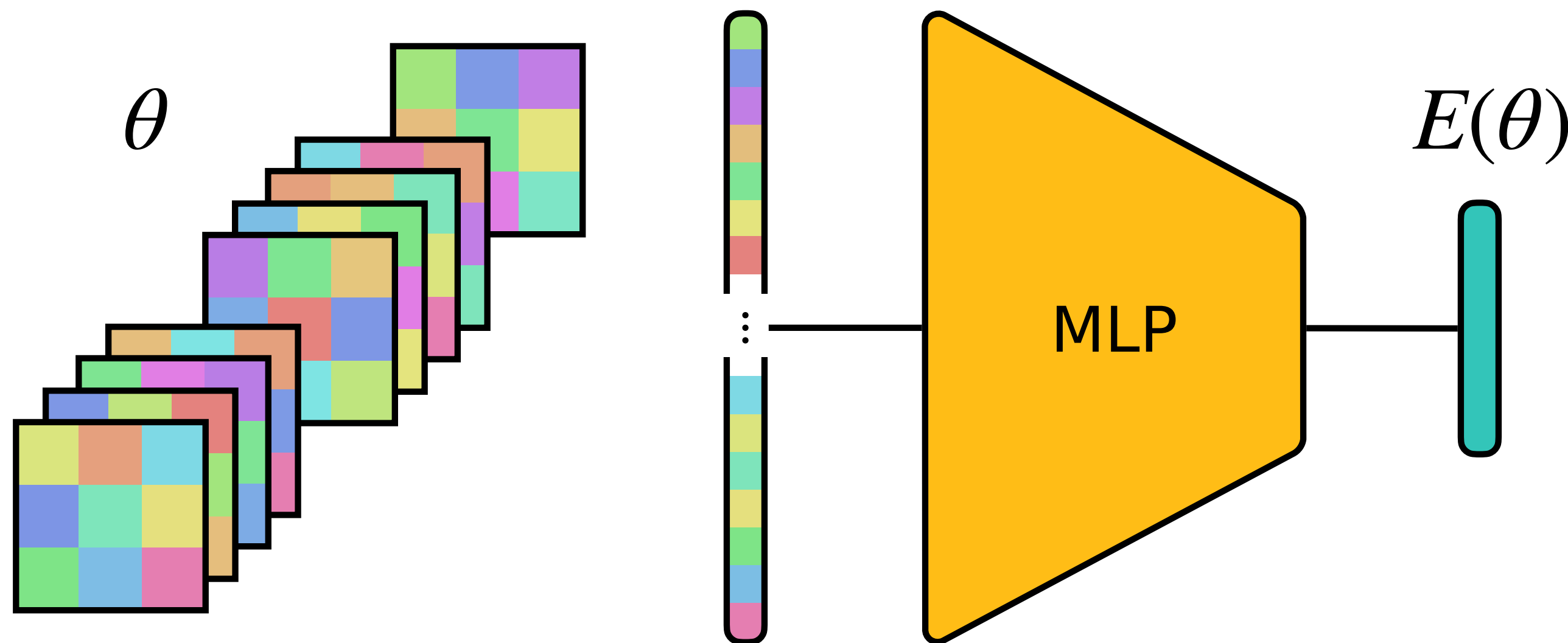


**compare:**

*Unterthiner et al., 2020, "Predicting Neural Network Accuracy from Weights"*

# Flattened Weights & MLP (Mechanistic)

- Flatten all weights into one big vector and feed into MLP
- Not invariant to hidden neuron permutation 🙄
- Universal 👍



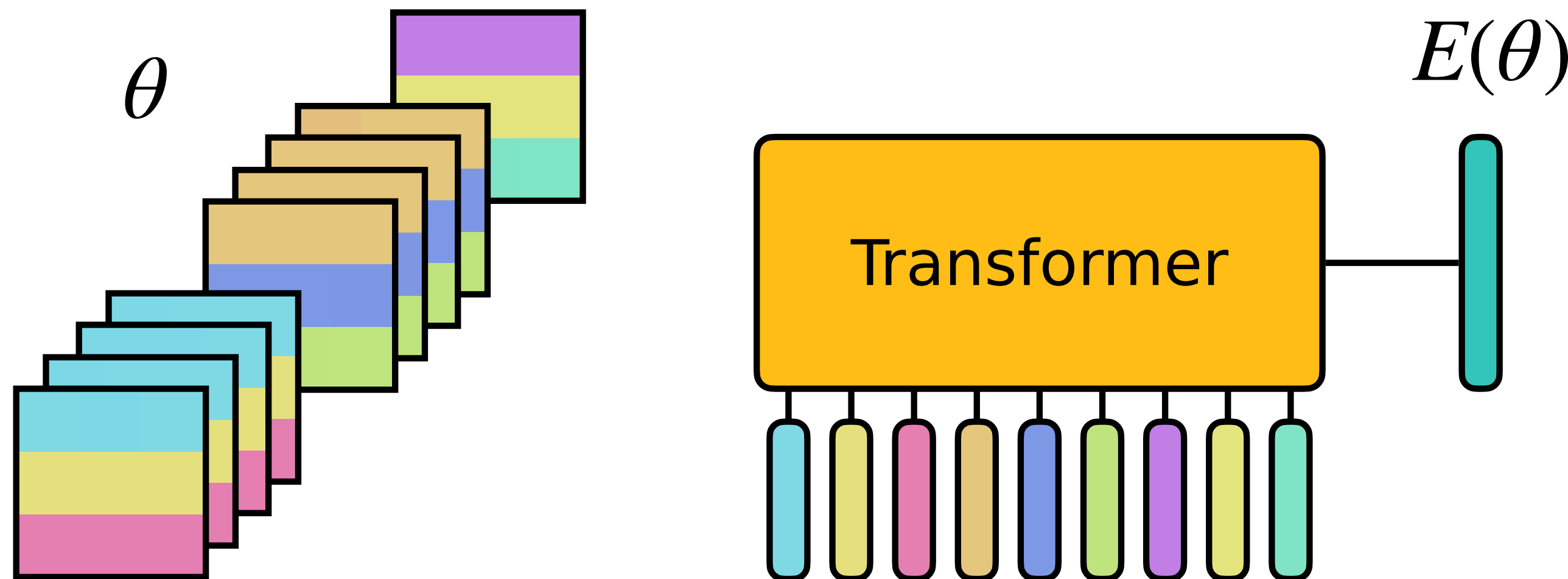
**compare:**

*Faccio, Kirsch, Schmidhuber, 2020,  
"Parameter-Based Value Functions"*

*Herrmann, Kirsch, Schmidhuber, 2022,  
"Learning One Abstract Bit at a Time  
through Self-Invented Experiments  
Encoded as Neural Networks"*

# Parameter Transformer (Mechanistic)

- Feed neuron weight vectors as sequence to a transformer model
- Not invariant to hidden neuron permutation 🙄
- Universal 👍

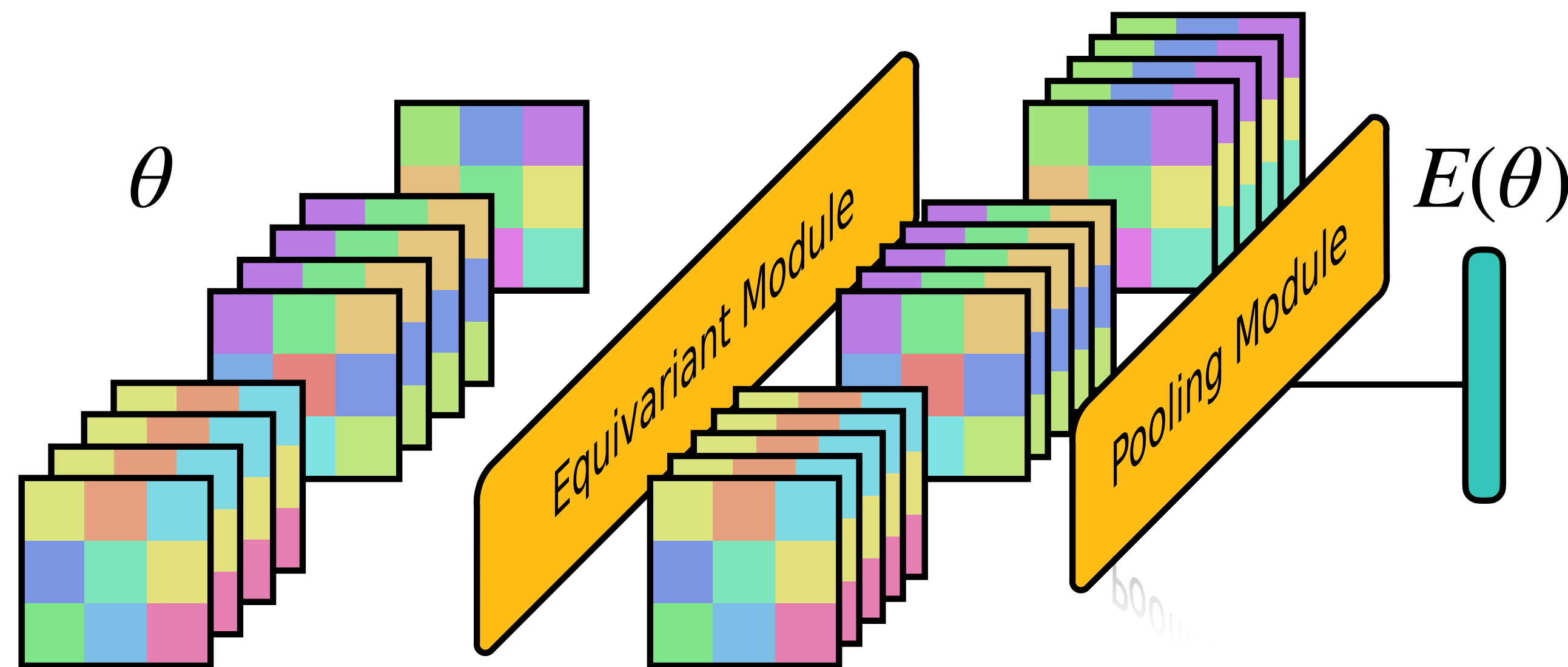


**compare:**

*Schürholt, Kostadinov, Borth, 2021,  
"Self-Supervised Representation  
Learning on Neural Network Weights  
for Model Characteristic Prediction"*

# Deep Weight Space Net (Mechanistic)

- Equivariant weight processing modules followed by invariant pooling layer
- Invariant precisely to hidden neuron permutation 👍
- Universal 👍



**compare:**

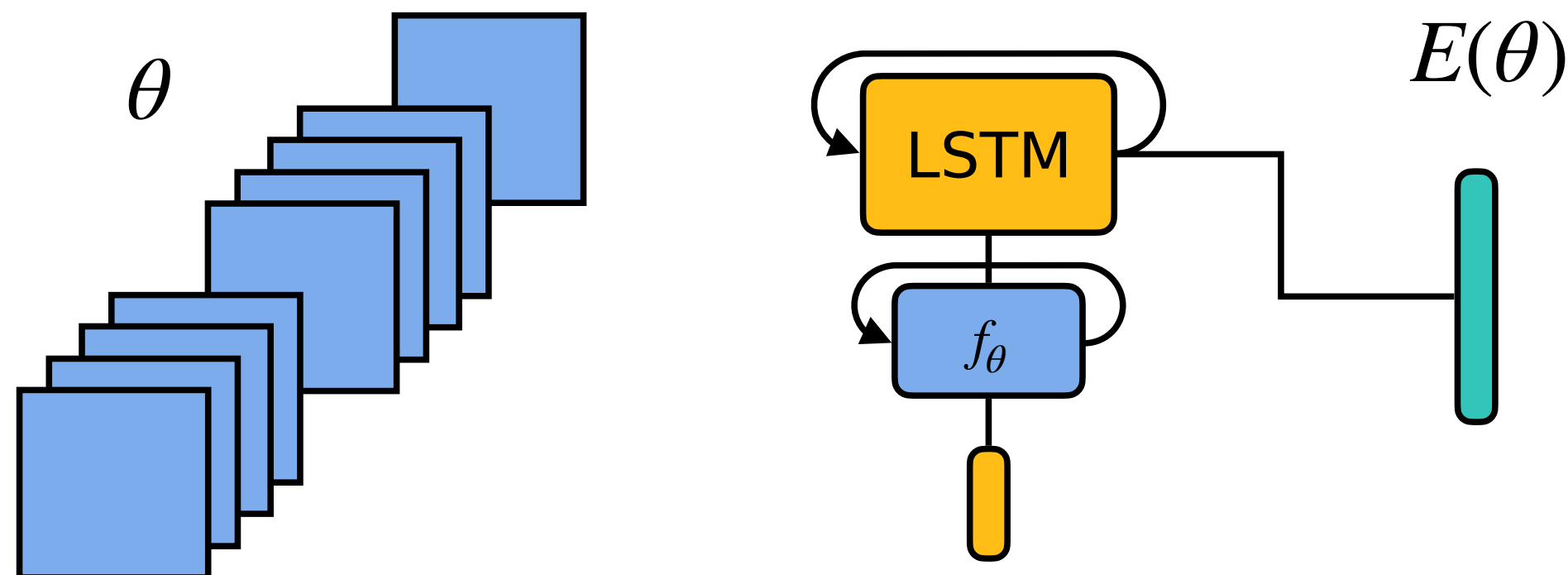
*Navon et al., 2023, "Equivariant Architectures for Learning in Deep Weight Spaces"*

*Zhou et al., 2023, "Permutation Equivariant Neural Functionals"*

*Kirsch, Schmidhuber, 2020, "Meta Learning Backpropagation and Improving it"*

# Non-Interactive Probing (Functionalist)

- Fixed but learnable probing sequences are given to  $f_\theta$
- Based on the corresponding probing outputs,  $E(\theta)$  is computed
- Invariant to everything that leaves  $f_\theta$ 's functionality intact 👍
- Not fully universal 🙅



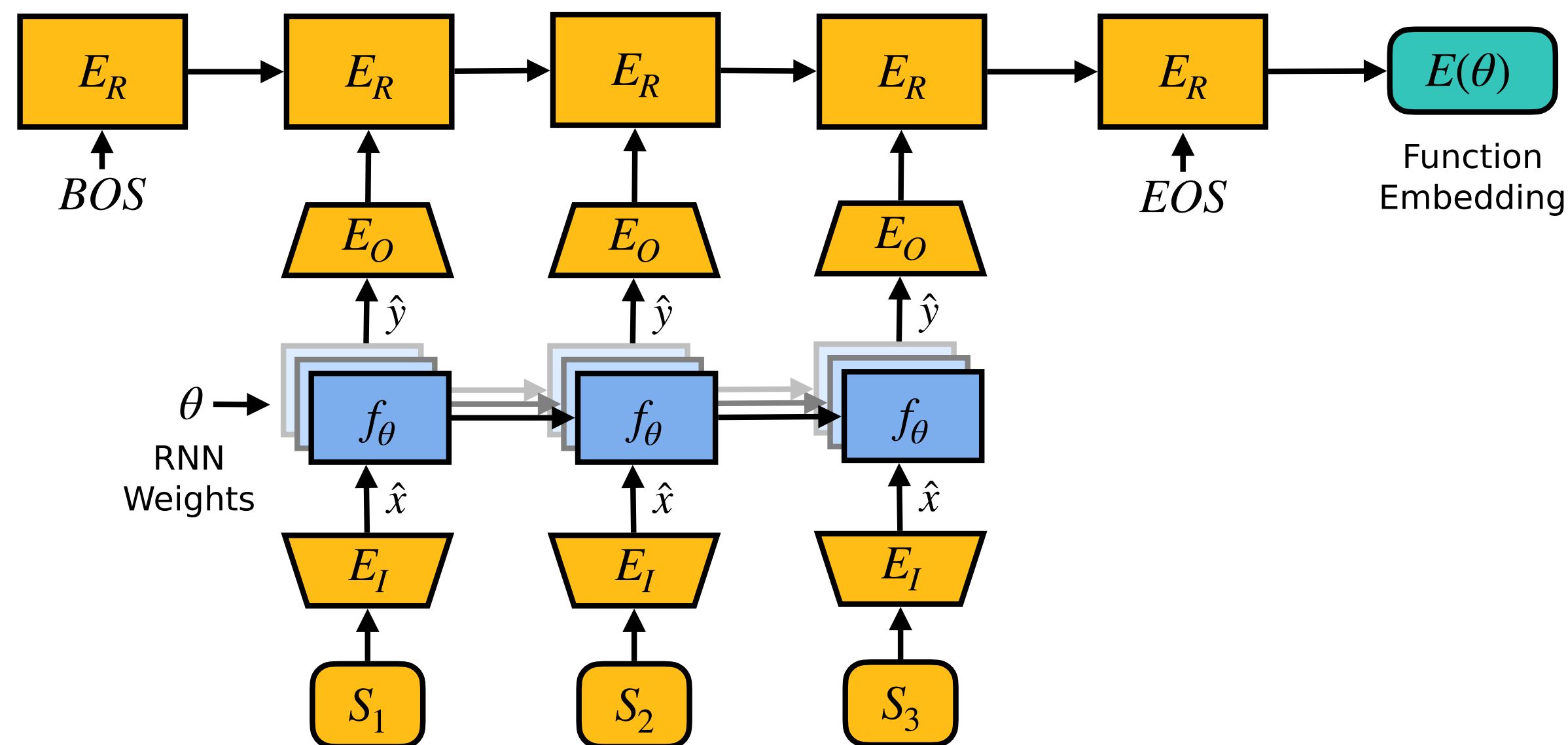
**compare:**

*Harb et al., 2020, "Policy Evaluation Networks"*

*Faccio et al., 2022, "Goal-Conditioned Generator of Deep Policies"*

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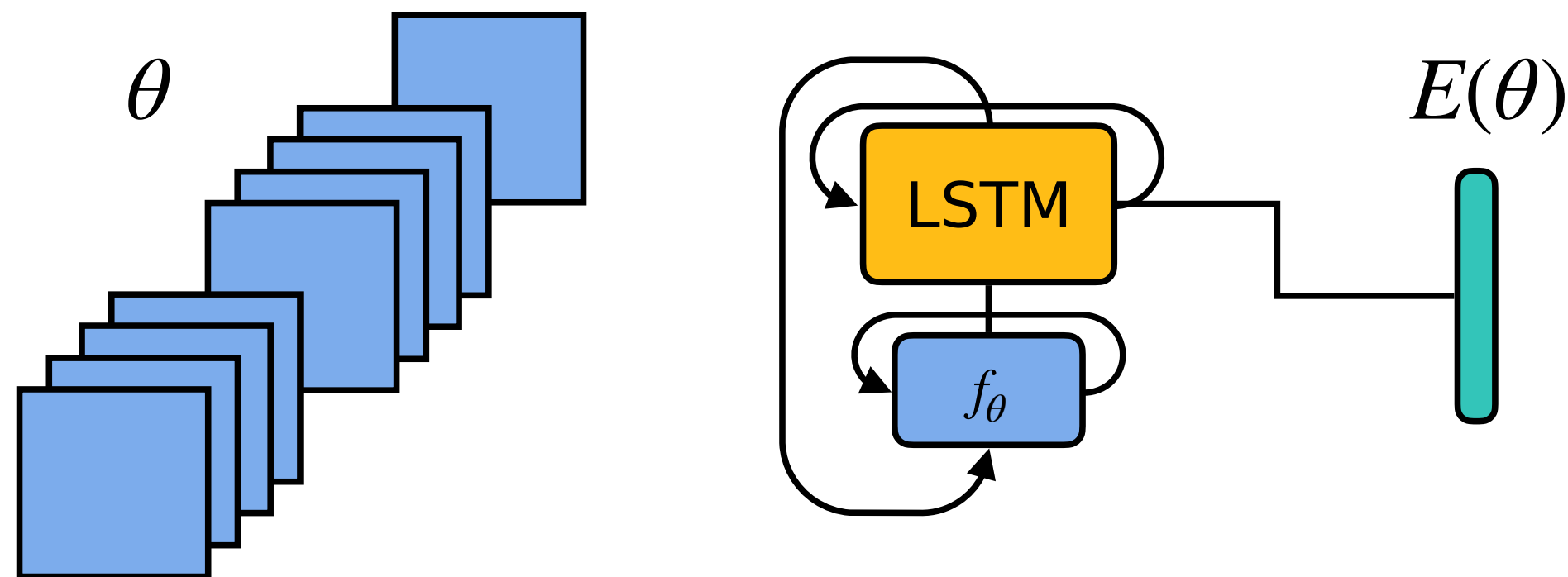
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# Interactive Probing (Functionalist)

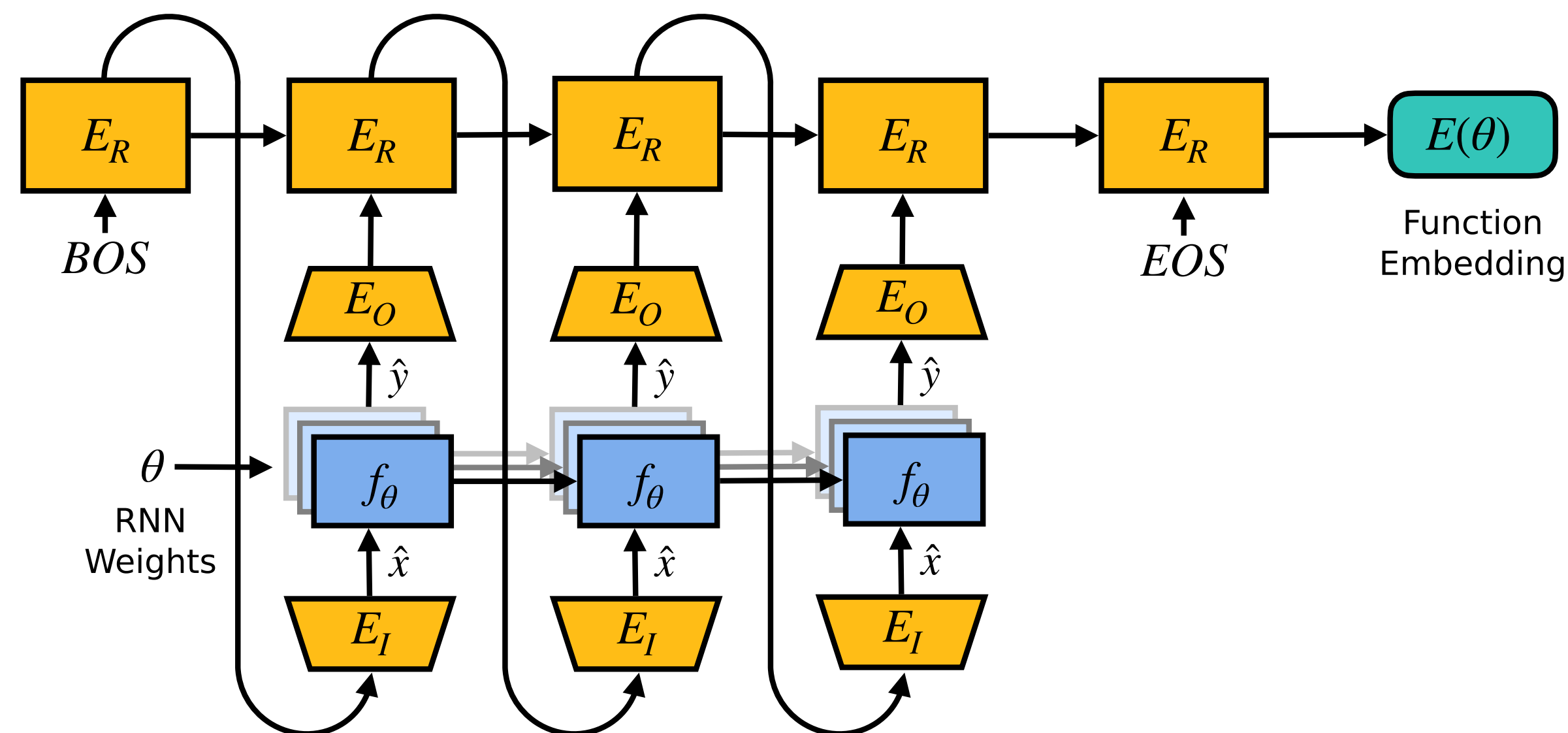
- Probing sequences are dynamically generated by core LSTM
- Next probing inputs depend on  $f_\theta$ 's response to all previous probing inputs
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**compare:**  
*Schmidhuber, 2015, "On Learning to Think"*

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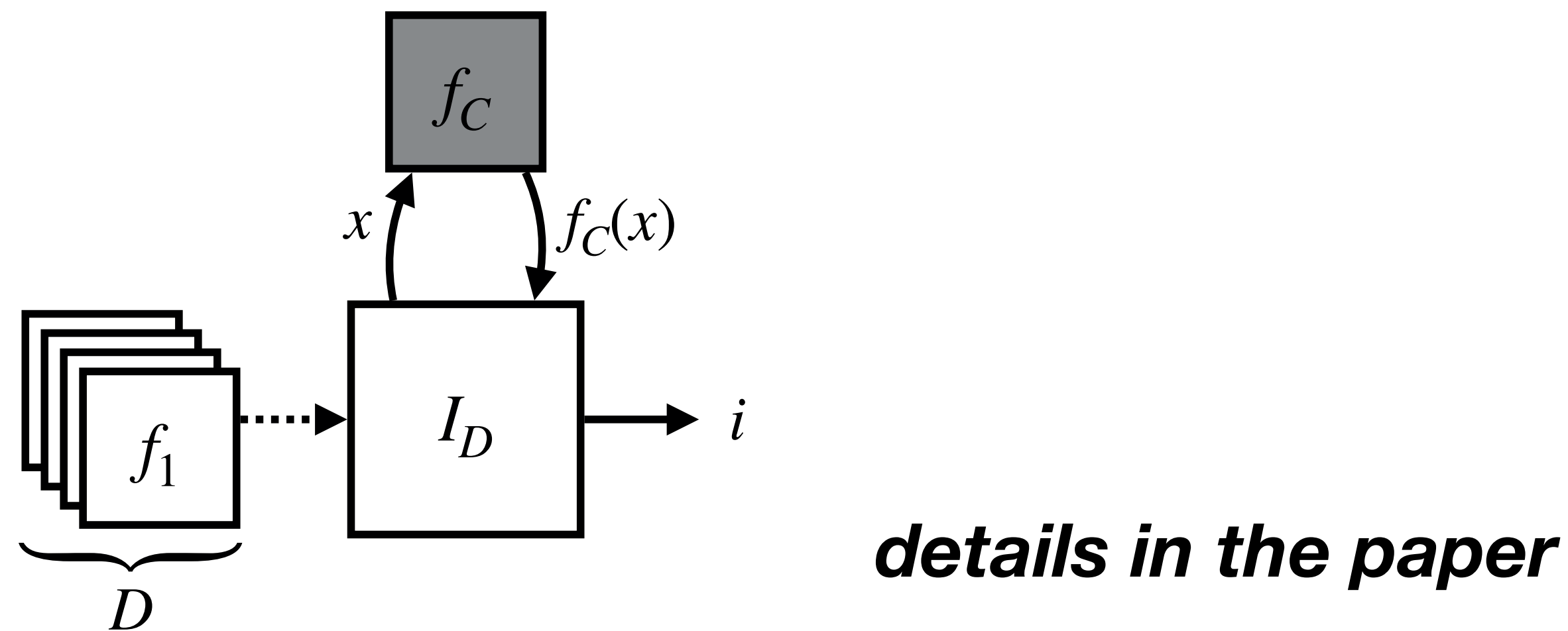
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# Functionalist Approach: Theoretical Results

*What's the difference between interactive and non-interactive probing?*



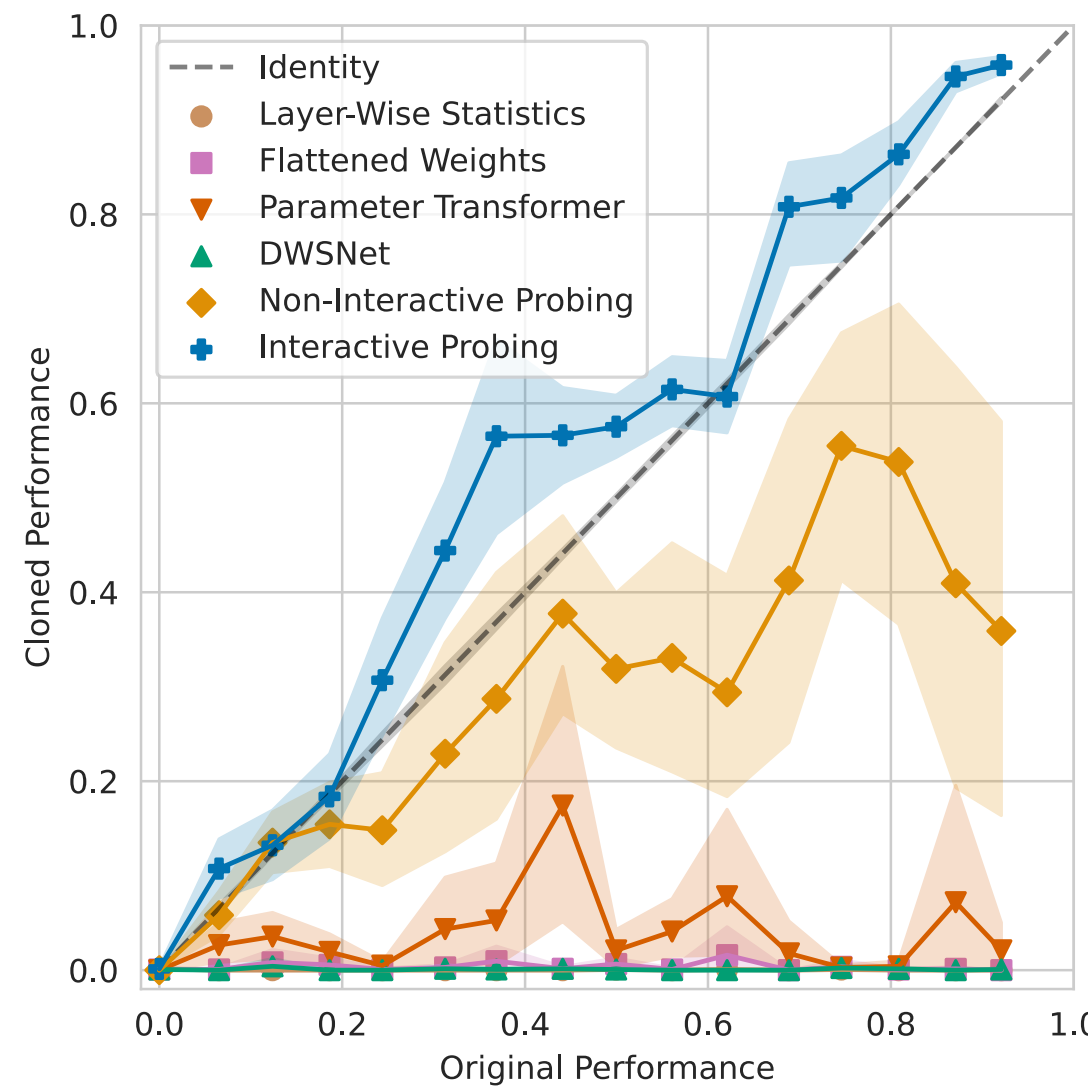
## Results

- General upper bound for required interactions is the same
- In certain settings, interactive probing is exponentially more efficient

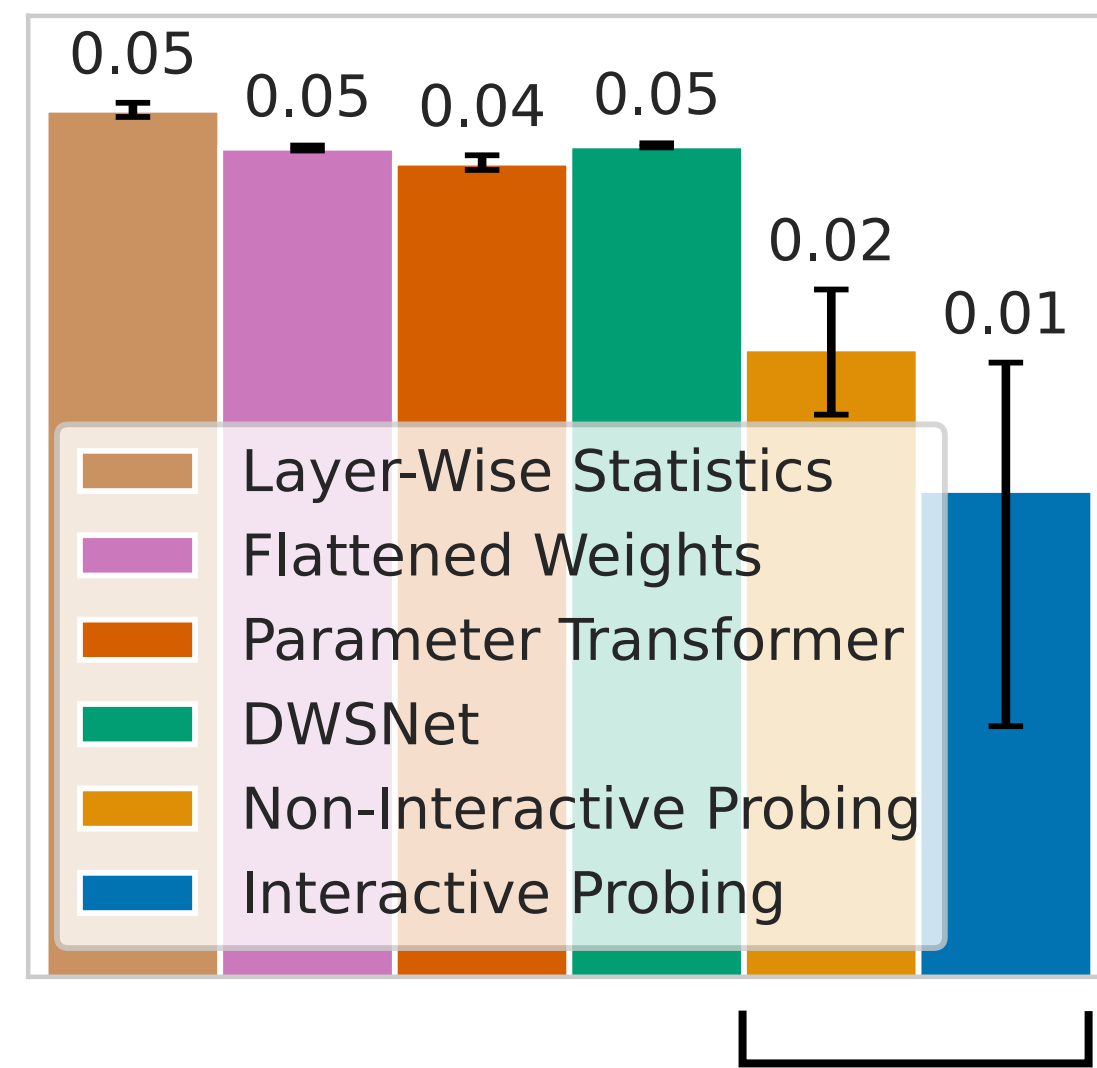
# Pre-Training Results

## Formal Languages

*Original vs. Emulated*



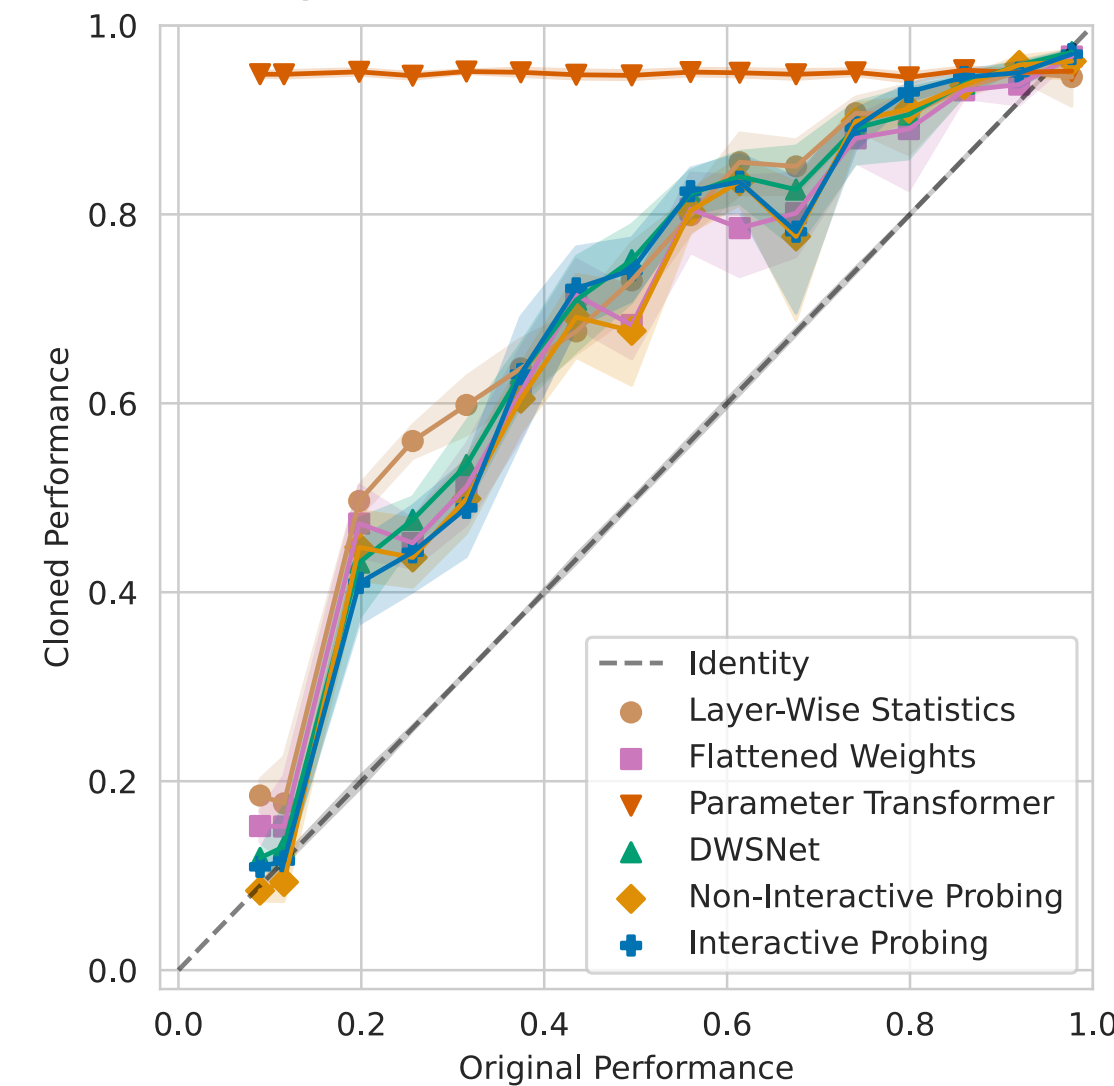
*Validation Loss*



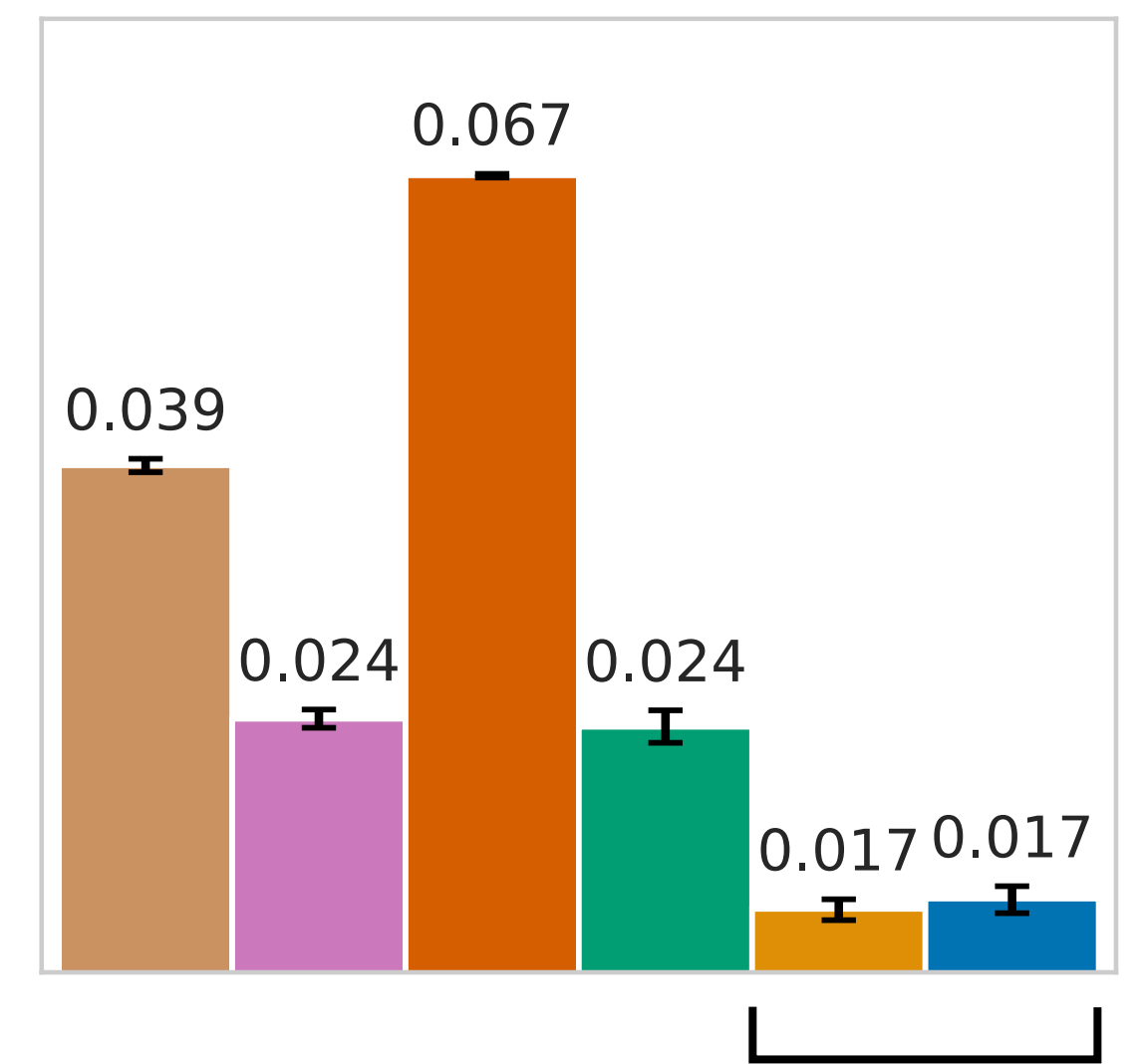
**Functionalist**

## Sequential MNIST

*Original vs. Emulated*



*Validation Loss*

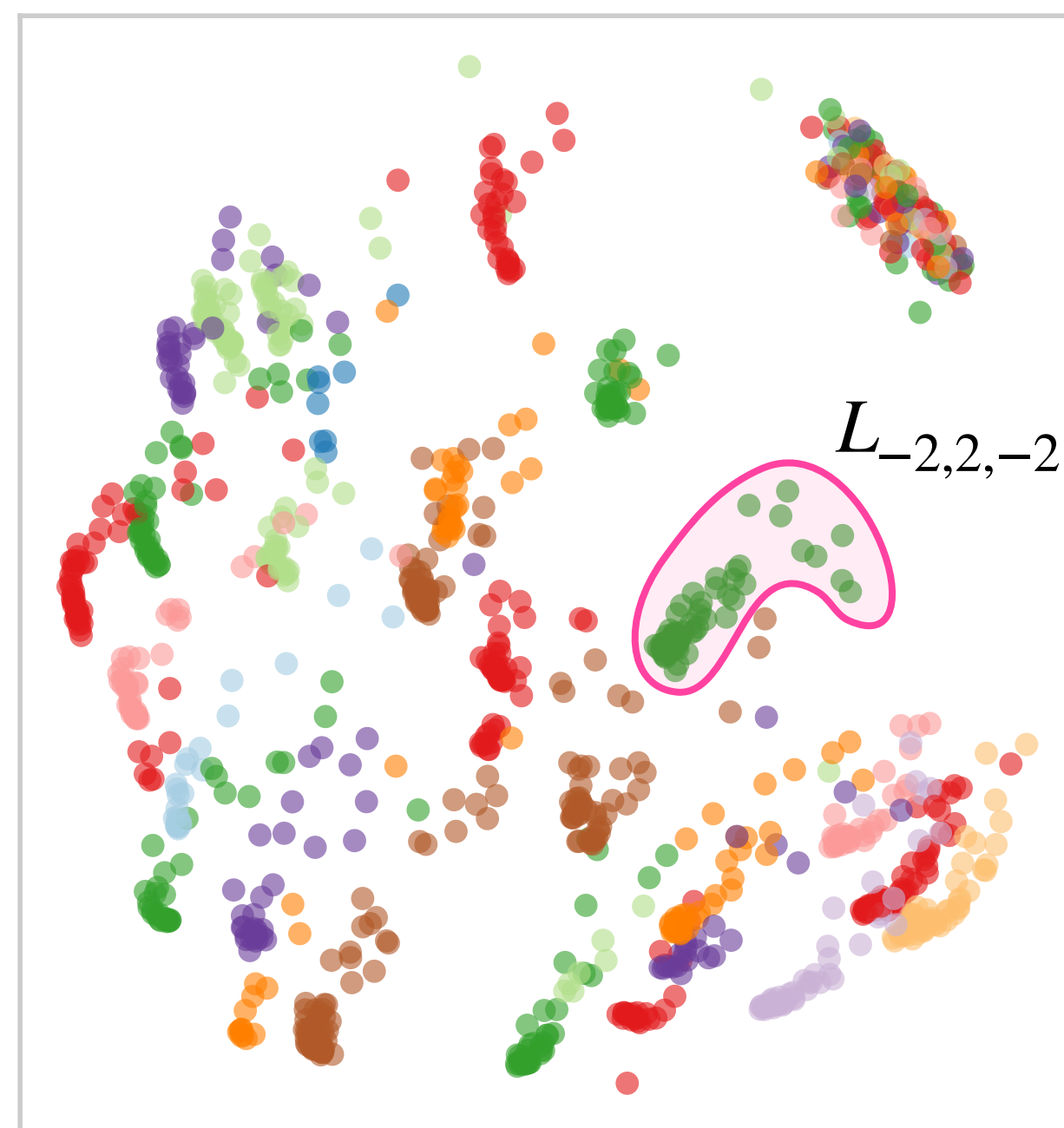


**Functionalist**

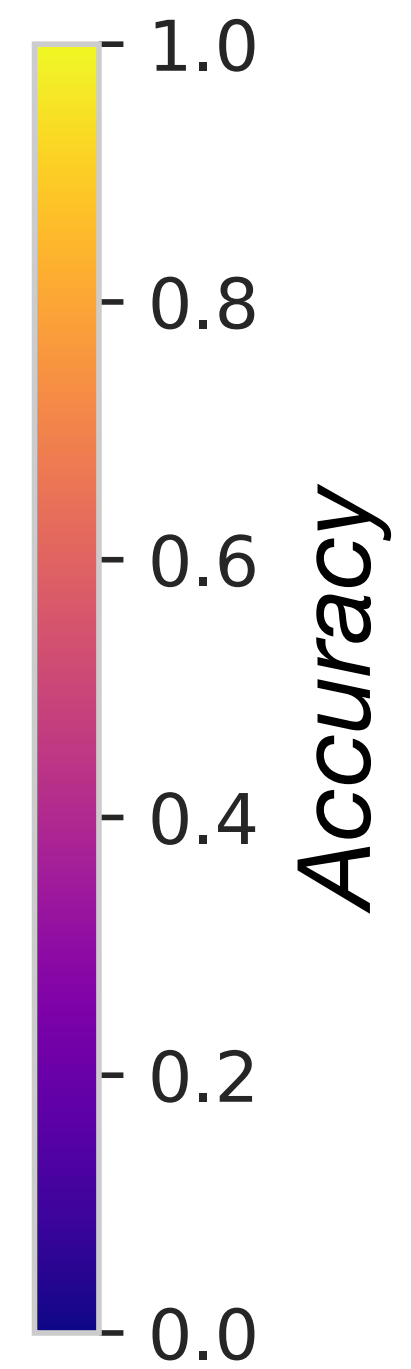
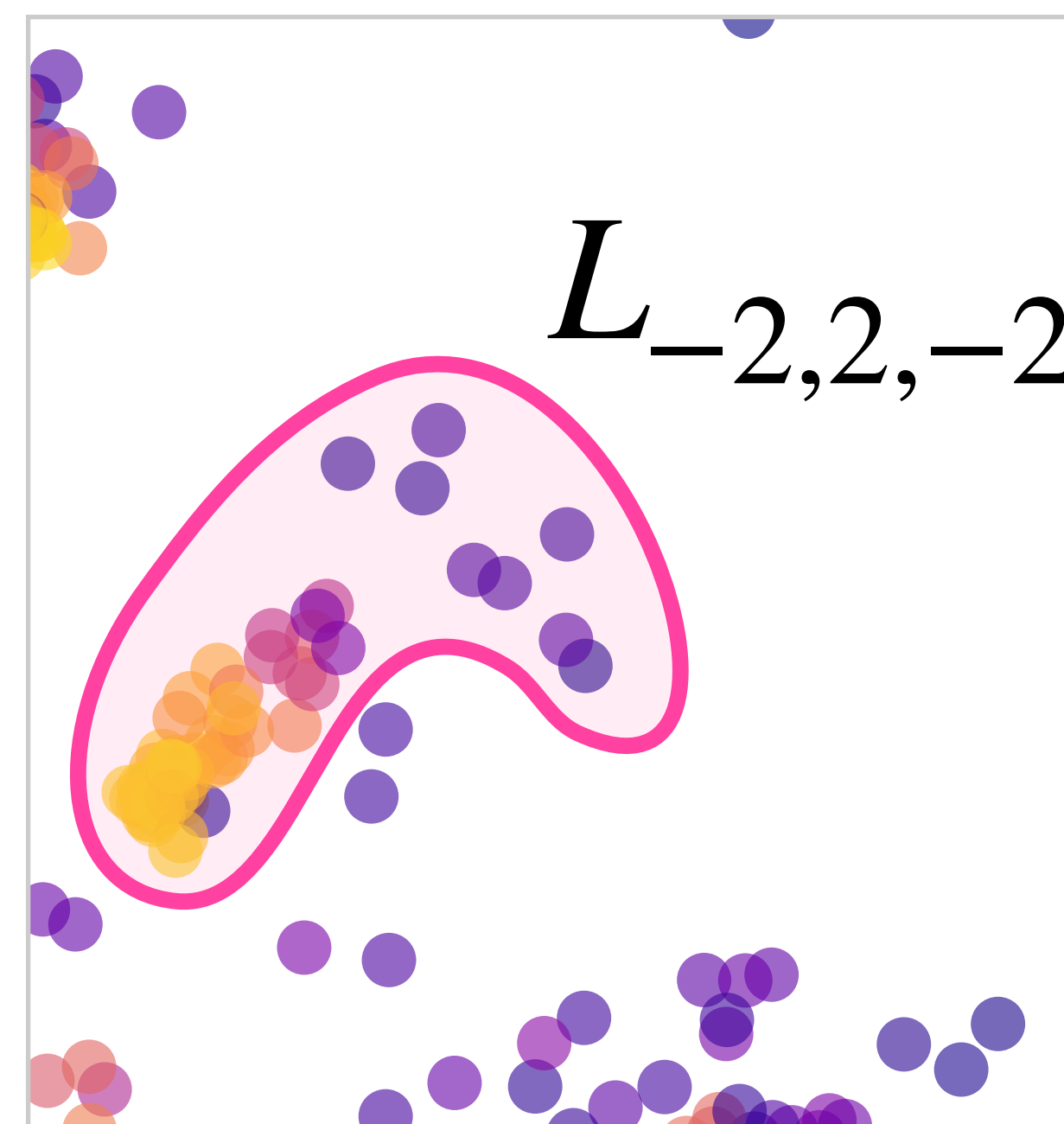
# Learned Embedding Spaces

Formal Languages

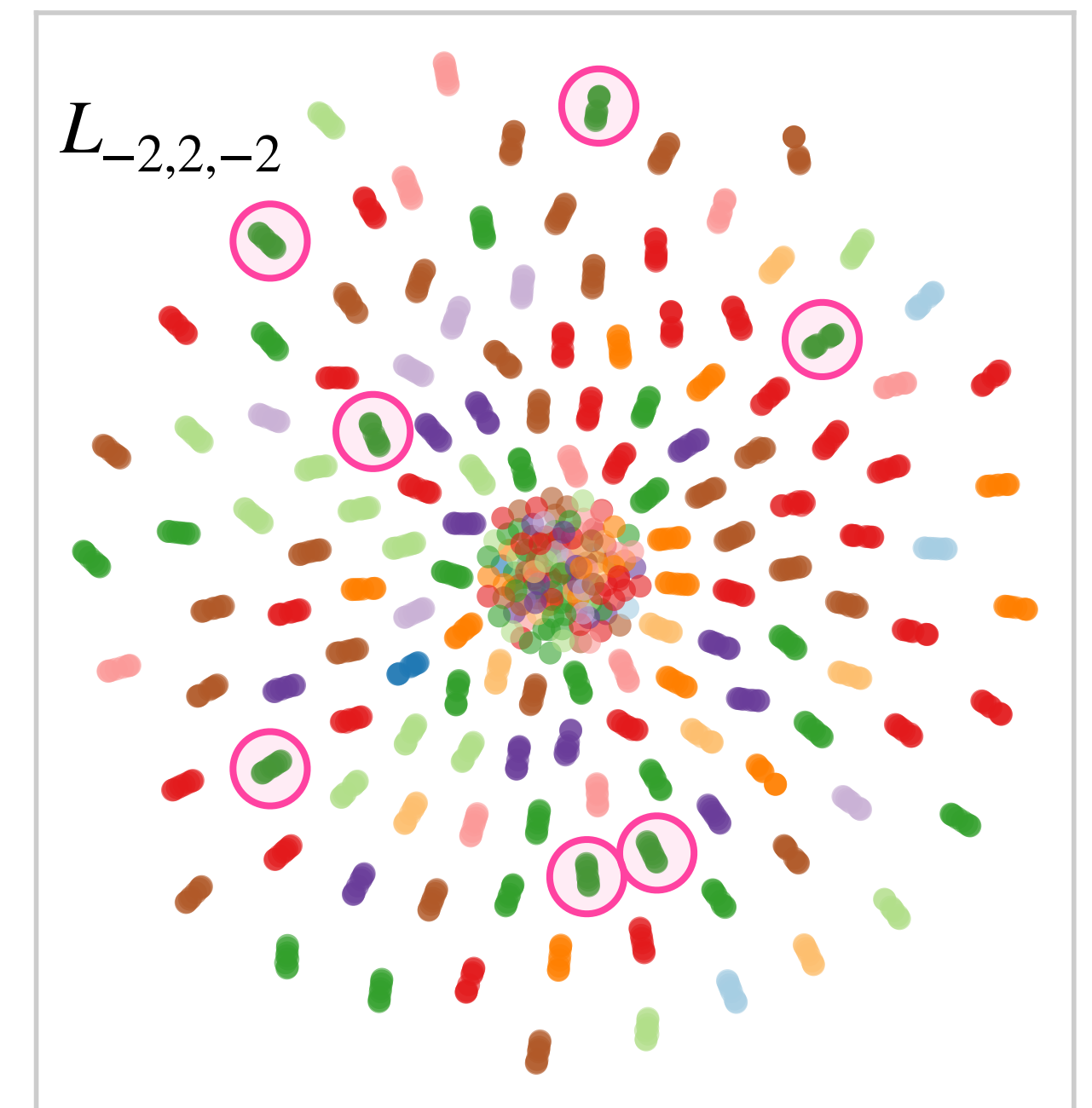
*Interactive Fingerprinting Embeddings (PCA)*



Task / Language



Comparison:  
*t-SNE of weights*



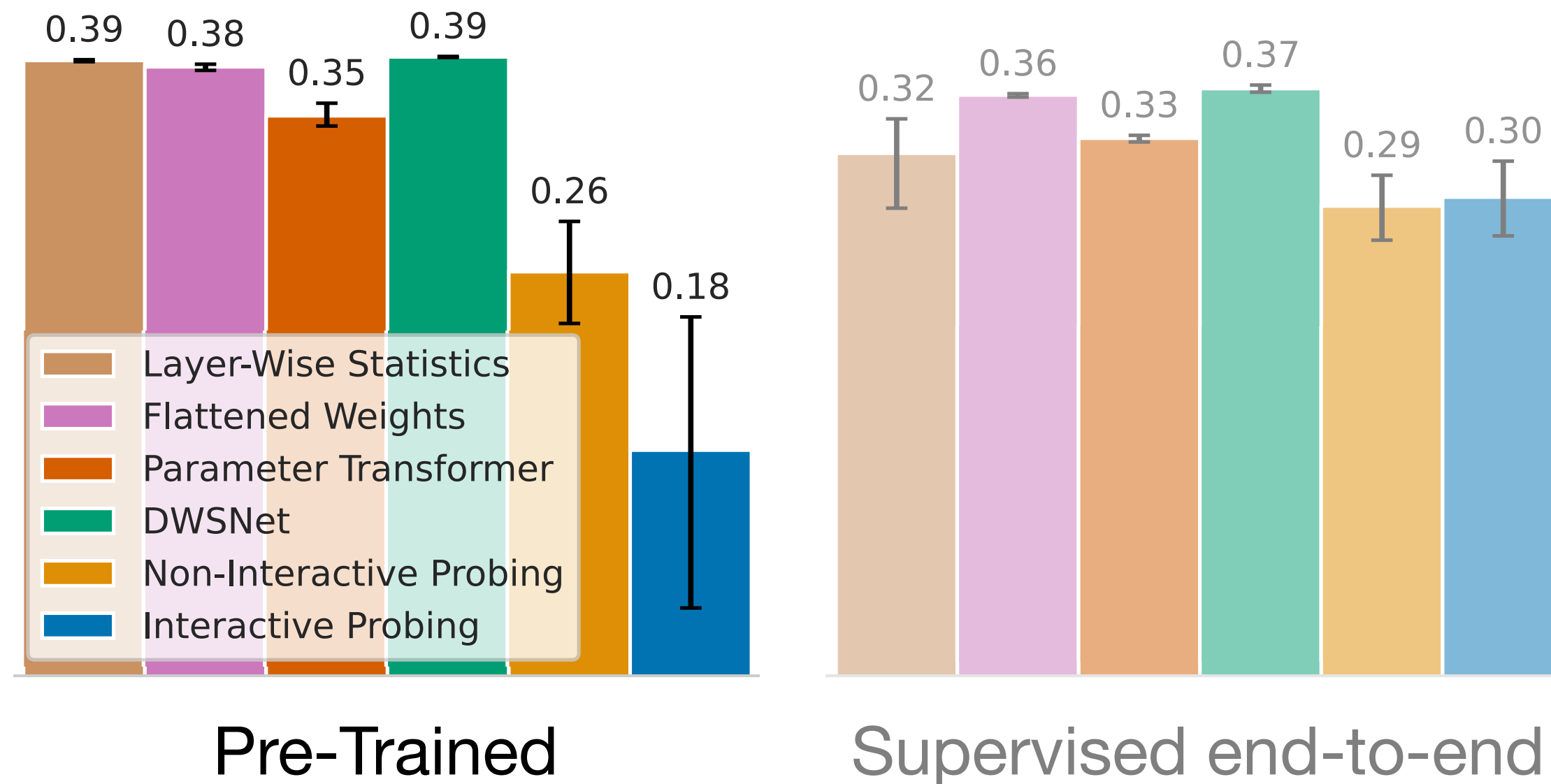
Task / Language

# Downstream Results

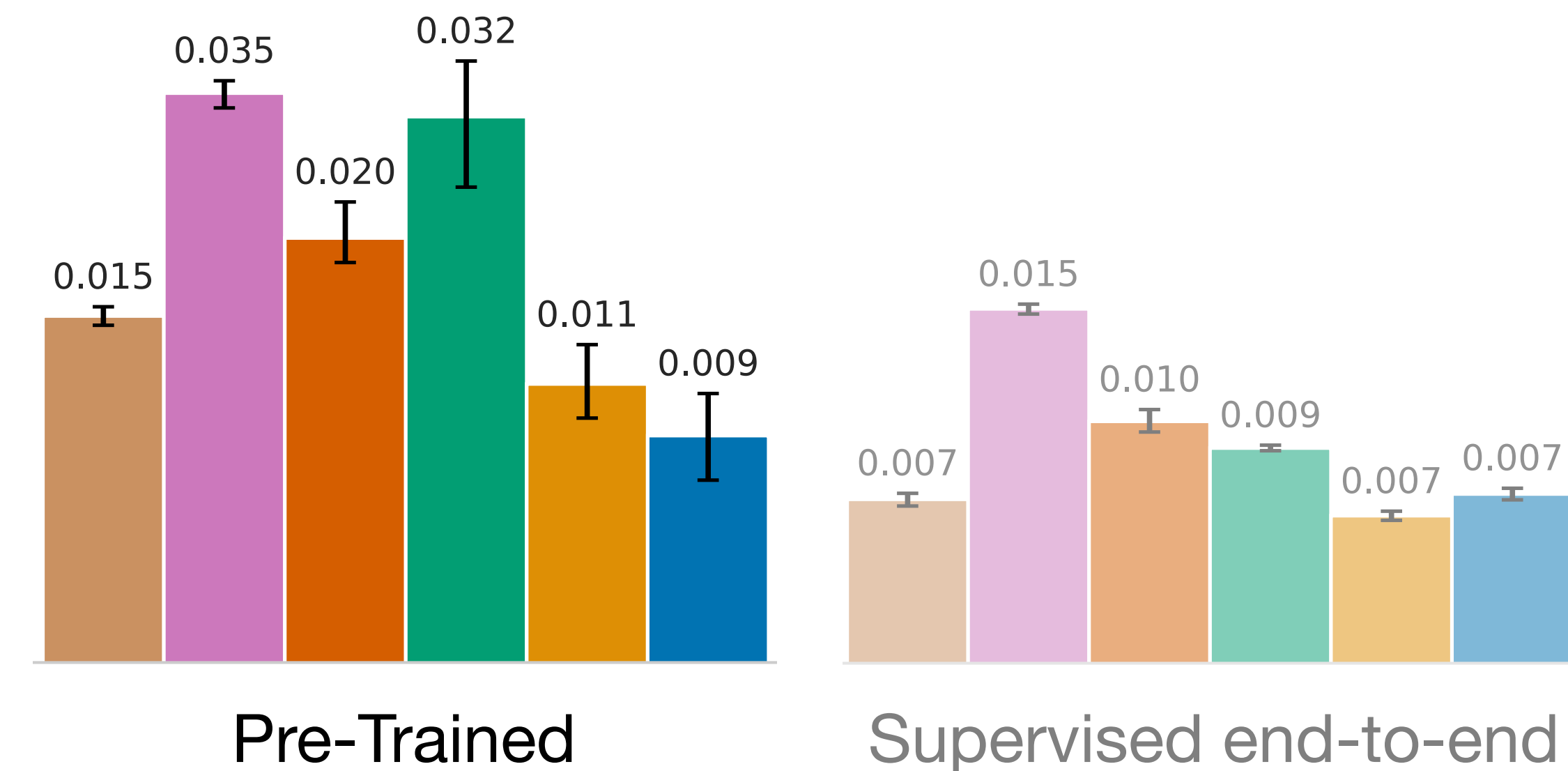
Predictor MLP on top of learned representations

Formal Languages

*Task prediction loss*



*Accuracy prediction loss*



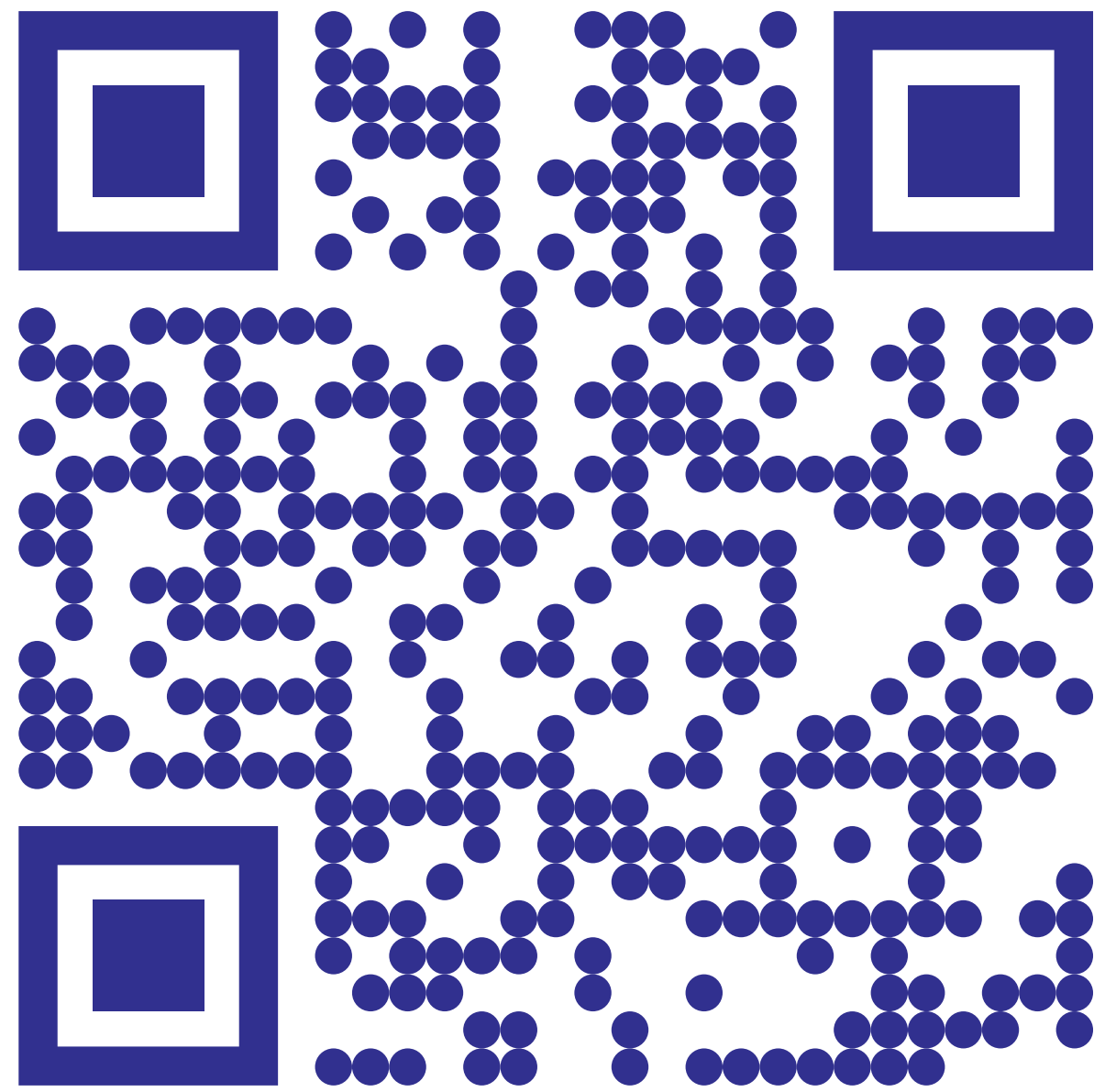
***more results in the paper***

# Conclusion

## Learning RNN Weight Representations

- Two RNN ‘Model Zoo’ datasets
- Emulation-based pre-training method
- Distinction between **mechanistic** and **functionalist** weight encoders
- Two novel functionalist encoder types
- Comparison of six different RNN weight encoder architectures

*Functionalist encoders are superior at complex tasks*



*Paper*

**Thank you!**



*Code & Datasets*